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A STUDY OF INDEPENDENCE BETWEEN SUPPLY ECHELONS

by

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ABSTRACT:

The Ships Supply Support Study, a study of supply support to the ships of the United States Navy under the direction of the Chief of Naval Operations, assumes that the availability of an item at a given echelon is independent of its availability at other echelons. A study of the validity of that critical assumption is made. A model of a multi-echelon supply support system which provides a history of the daily status of each entity in the system and a stock profile at each echelon is developed. Using randomly generated demands and current replenishment rules for a representative number of items generated at representative activities at each echelon, gross supply availabilities and conditional availabilities are calculated, and statistical tests of the assumption are made. Mean supply response times are computed using both the gross supply availabilities and the conditional availabilities to illustrate the impact of dependence among echelons.

The tests indicate that the availability of an item at a given echelon can in many cases be strongly dependent on its availability at the other echelons. When this is so, the cost in terms of an increase in mean supply response time can be substantial. However, the independence hypothesis is not rejected whenever the gross supply availabilities throughout the system are either very nearly unity or zero.

Prepared by:

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TABLE OF SYMBOLS AND ABBREVIATIONS

a_i	- Conditional Availability at Echelon i
A_i	- Marginal Availability at Echelon i
A_o	- Operational Availability
AFS	- Combat Stores Ship
AIMS	- Afloat Inventory Management Simulator
COG	- Material Cognizance Class
COSAL	- Consolidated Shipboard Allowance List
DSA	- Defense Supply Agency
FILL	- Fleet Issue Load List
ICP	- Inventory Control Point
MADT	- Mean Administrative Delay Time
MLSF	- Mobile Logistics Support Force
MLDT	- Mean Logistic Delay Time
MSRT	- Mean Supply Response Time
MTBF	- Mean Time Between Failure
MTTR	- Mean Time to Repair
Q	- Reorder Quantity
R	- Reorder Level
S^4	- Ships Supply Support Study
χ^2	- Chi-squared Test Statistic

1. INTRODUCTION

An in-house study of supply support to the units of the United States Navy, the Ships Supply Support Study (S^4), was commissioned by the Chief of Naval Operations in August of 1971 [Ref. 5]. The purpose as stated in the study directive, was to "define, develop and propose an automated method by which supply support dollar outlays may be related to fleet capability." This goal was to be achieved by developing and studying two Supply System Simulators, each a combination of several analyzers and simulators designed to explore the relationships between funds available for various logistics purposes and the response time that can be achieved by those funds. In particular, the simulators are designed to provide answers to questions which seek to determine the relationships between system effectiveness and dollar outlays for supply support such as:

A. What is the effect on operational availability if requisition response time is either increased of decreased?

B. What would happen if the Mobile Logistic Support Force were relieved of all end-use requisition functions?

C. What would be the effect on requisition response time if the Consolidated Shipboard Allowance List (COSAL) were increased or decreased by a specified percentage?

D. What would happen if budgets were increased or decreased by a specified amount at an Inventory Control Point (ICP)?

E. Are increases in availability more cost-effective than decreases in throughput times?

F. Where should extra monies be allocated if budgets are increased or, similarly, where should cuts be made if budgets become tighter?

In order to accomplish the assigned task the Sixth Fleet and its external supply support was chosen as the locus of the study to simplify administrative and data collection problems. In addition, some ships of the Sixth Fleet were excluded to further simplify the problem.

Basically, the supply support system for the Sixth Fleet is similar to that of the other fleets. A requisition originating at a ship is either satisfied at the ship itself or passed on to the next higher echelon of support. Screening of "sister" ships occurs whenever the requisition is for an emergency or high priority item. The higher support level is either the Mobile Logistics Support Force (MLSF) if the part is carried on the load list (FILL) of the MLSF, or to the Naval Supply Center (Norfolk, for the Sixth Fleet) if the part is not a FILL item. As with the first screening level in the case of high priority items, other deployed units in the Sixth Fleet may be screened by MATCONOFF if the MLSF cannot supply the needed equipment. If the requisition is still unsatisfied at the Naval Supply Center, it can flow from this echelon to either a Navy Inventory Control Point (ICP), a Defense Supply Center, or the General Services Administration warehouse depending on the nature of the item. The highest echelon, assumed to have 100% availability, is the manufacturer. The alternative actions available at various levels are indicated by the flow chart in Figure 1.

The computer model developed by the S^4 group examines the ratio of the number of requisitions filled to the number of requisitions received, called gross supply availability, at each echelon. These availabilities are then used along with estimates of throughput times

SUPPLY ECHELONS

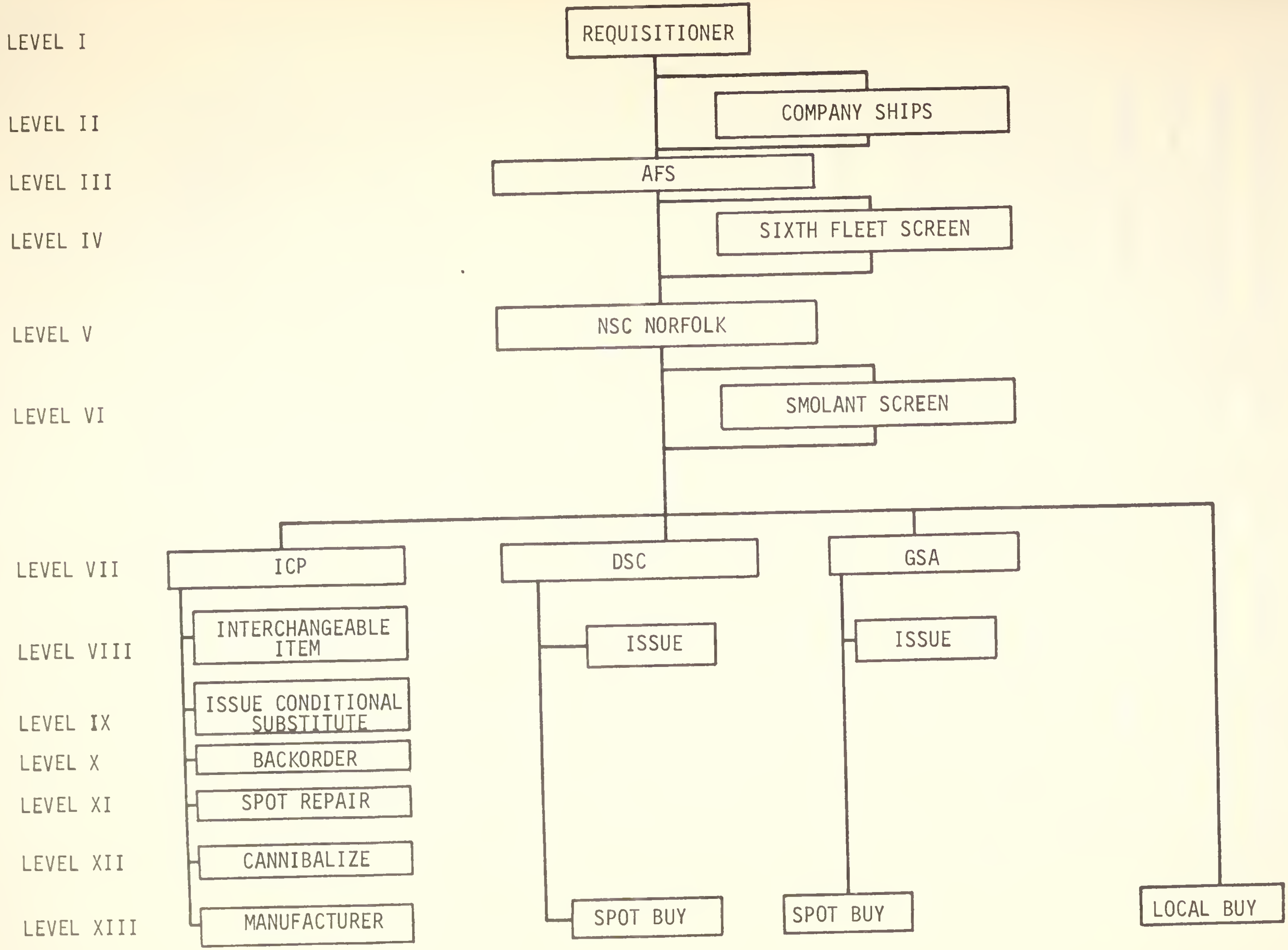


FIGURE 1

to produce estimates of the average response times. Briefly, the first simulator, Mark I, developed by the S^4 group consists of five inventory simulators and a single synthesizer. There is a shipboard simulator wherein actual demands as experienced over an 18 month period are placed against the ship's COSAL. Actions to satisfy the demand are taken and receipts of the requisitioned stock are scheduled. Additionally, there is a simulator for the Mobile Logistics Support Force which is virtually identical to the shipboard simulator, a stockpoint simulator used to forecast the probability that NSC Norfolk can satisfy a requisition for 9-cog material submitted by a requisitioner, and two ICP simulators to account for minor variations in requisition processing between the Electronic Supply Office and Ships Parts Control Center. The stockpoint simulator and the ICP simulators are all single-item simulators in which demands are generated randomly from computed means and variances in each. The synthesizer receives the output from each of the simulators to estimate average requisition response time by material cognizance class and the inventory and workload associated with a given response time.

Within the simulators and synthesizer, a given requisition is not moved through the successive echelons until it is satisfied. Instead, new requisitions are introduced at each echelon, and they are either satisfied or killed at that echelon. A single number about each echelon, the gross supply availability, is transmitted to the synthesizer. The sole agent making requisitions at the various echelons compatible is the fact that all simulator inputs are taken from observations at the various levels of the Naval Supply System.

In developing the simulators and the synthesizer the study group assumed that the availability of an item at a given echelon is independent of its availability at the other echelons. In effect, this assumption justifies the type of modeling of the multi-echelon Naval Supply System which the study group has done. The purpose of this paper is to report the findings of a study devoted to testing the validity of that critical assumption, and to investigate the impact of the assumption on the estimates of mean requisition response times.

2. SUPPORT EFFECTIVENESS: MEAN SUPPLY RESPONSE TIME

In order to evaluate the various cost trade-offs that can be analyzed in the complex multi-echelon Naval Supply System, a measure of effectiveness must be chosen. Operational availability is used for this purpose in the Ships Supply Support Study. The operational availability of a component is defined as the ratio of the mean time between failure (MTBF) to the sum of MTBF, the mean time to repair (MTTR), and the mean logistic delay time (MLDT):

$$A_o = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR} + \text{MLDT}} \tag{2.1}$$

Mean logistic delay time is further partitioned into mean administrative delay time (MADT) and the mean supply response time (MSRT). The latter is defined to be the average amount of time required to get the needed unit or units into the hands of a mechanic aboard the requisitioning ship. If the administrative delay time is absorbed into the time to repair, operational availability becomes:

$$(2.2) \qquad A_o = \frac{MTBF}{MTBF + MTTR + MSRT}$$

Earlier studies have indicated that MSRT is substantially larger than the combined value of the mean administrative delay time and the mean time to repair. Because MSRT generally dominates the other factors it is the driving force in the equation for operational availability, and it is clearly that factor through which the supply system can make its greatest contribution towards increasing operational availability. It is important to know how improvements can be made in MSRT.

The magnitude of MSRT is itself a function of the structure and behavior of the Supply System. Suppose a system has n echelons with the lowest echelon being the ship storeroom and the n^{th} echelon being the manufacturer of the part. Whenever a unit fails, a replacement is supplied from the ship's storeroom if the part is available; otherwise, a requisition is sent forward through the successive echelons until it is either supplied or manufactured. The mean supply response time is the sum of the response times of each echelon weighted by the fraction of total requirements that it satisfies. Mathematically,

$$(2.3) \qquad MSRT = a_1 t_1 + \sum_{j=2}^{n} a_j t_j \prod_{i=1}^{j-1} (1 - a_i)$$

where a_j = probability that the j^{th} echelon activity is able to satisfy an end-use requisition given that it cannot be satisfied by a lower echelon.

t_j = time from the mechanics need for a unit of material until his receipt from the j^{th} echelon activity.

Note that the weight given to the time t_j in (2.3) is $a_j \prod_{i=1}^{j-1} (1 - a_i)$.

This represents that fraction of material needed for repair which is available at the j^{th} echelon and not available at any of the lower echelons. One should recognize that a_i, a conditional probability or conditional availability, is not the gross supply availability. Rather, a_i refers to the ability of the i^{th} echelon to supply those parts which lower echelons stocked, but which were temporarily out of stock, and those parts which lower echelons did not stock. It is important to realize that the inventory of each echelon of supply in the Navy, serves two missions:

(1) it resupplies the bins and storerooms of each lower echelon directly or indirectly in an attempt to maintain the availability of that echelon, and

(2) it supplies items to the end user which are not carried by any lower echelon or not available at any lower echelon.

If the shipboard storeroom carried a complete range and sufficient depth of every item needed for corrective maintenance, and if the basic resupply mission of all other echelons were perfectly executed, the higher echelons would not need to perform the second mission. Unfortunately these capabilities are not economically feasible and, as a result, the dual missions of the echelons must be considered. The figure which represents the ability of an echelon to satisfy the second mission, a_i, is possibly different from the gross supply availability which represents the ability of an echelon to satisfy both of its missions. Thus, theoretically, it is only at the ship level and the manufacturer's level that the availability to satisfy immediate requirements is equal to the conventionally measured gross supply availability.

It was assumed in the Ships Supply Support Study that the availability of an item at a given echelon is independent of its availability at other echelons. Let the gross supply availability at echelon i be denoted by A_i. Then the assumption, in effect, says that the A_i's can be used in place of the conditional availabilities a_i in Equation 2.3. The S^4 group assumed that all end-use requisitions and only end-use requisitions are assigned issue priority group one or two (IPG I or IPG II) and stock replenishments or routine resupplies are IPG III. To justify the assumption of independence of echelon availabilities, the S^4 group reported that an analysis of historical supply availability by IPG revealed that:

(a) The difference between wholesale supply availability (ICP availability) for IPG III requisitions within a cog symbol and the combination of IPG I and IPG II requisitions was never more than 2.8% points and was usually less than 0.7% points.

(b) The hypothesis that observed values of IPG I and IPG II requisition availabilities came from the same distribution as IPG III availabilities could not be rejected.

(c) The Sixth Fleet indicates that little rationing of stock, whereby units are set aside solely for the purpose of satisfying emergency requests, is performed aboard the AFS, the member of the Mobile Logistics Support Force of interest in this study.

These facts seem to lend support to the assumption of independence made by the S^4 group.

Nevertheless, let us now view the stock profile for a given item at echelon i as an alternating renewal process which takes the value unity if the item is in stock at time t and which is zero if that item is out of stock at echelon i.

$$(2.4) \qquad Z_i(t) = \begin{cases} 1 & \text{if stock is greater than zero} \\ 0 & \text{otherwise.} \end{cases}$$

It would reasonably be expected that whenever a given echelon is out of a given item the leadtime is short if the next higher support echelon is in stock, but it is long if the requisition must be satisfied at an activity several echelons removed. In the Naval Supply System a requisition would flow to a level several echelons higher only if all lower echelons were out of stock. Thus, long intervals of time in which each of several echelons are out of stock simultaneously would not seem unlikely. When this happens the availability suffers an adverse affect. Figure 2 illustrates a possible relationship among the processes $Z_1(t)$, $Z_2(t)$ and $Z_3(t)$ in a hypothetical four echelon supply system. ($Z_4(t)$ is assumed to be identically 1.)

From Figure 2 we see that short leadtimes occur whenever the next higher echelon is in stock but longer leadtimes are indicated if the next echelon is out of stock. Also, using Figure 2, we obtain the following estimates for the gross supply availabilities and the conditional availabilities:

$$A_1 = .69 \qquad a_1 = .69$$

$$A_2 = .74 \qquad a_2 = .45$$

$$A_3 = .72 \qquad a_3 = .33$$

In this particular example there are substantial differences between the numbers reported for gross supply availability and conditional availability at the second and third echelons. If the alternating renewal processes were independent processes the two availability figures would theoretically be identical. On the other hand, a comparison of the estimates of gross supply availability and conditional availability, and a study of the resulting impact on mean supply response times provide us with a rough quantitative measure of the importance of any dependence.

3. METHOD OF STUDY

A restatement of the purpose of this study is to test the hypothesis that the set of alternating renewal processes describing the stock profiles of each echelon is a set of independent processes and to quantify the effect of the dependence on mean supply response time if the hypothesis is rejected. In order to test the hypothesis, the stock profile of each echelon must be examined by calendar time so that all echelons are placed on a common time scale. This makes it necessary to follow each requisition, both for end use and resupply, throughout the supply system until the requisition is satisfied.

The most direct and possibly the most useful way to test the hypothesis would be to model the supply system mathematically and then determine analytically the degree of dependence as a function of the structure of the system, the interactions between the echelons, the process generating demands and the replenishment policies of each echelon. Unfortunately, attempts to obtain analytical solutions to mathematical models of even very simple multi-echelon supply systems have proven historically to be unsuccessful. This is particularly true if it is necessary to follow requisitions throughout the system. Thus, the computational implications of obtaining an

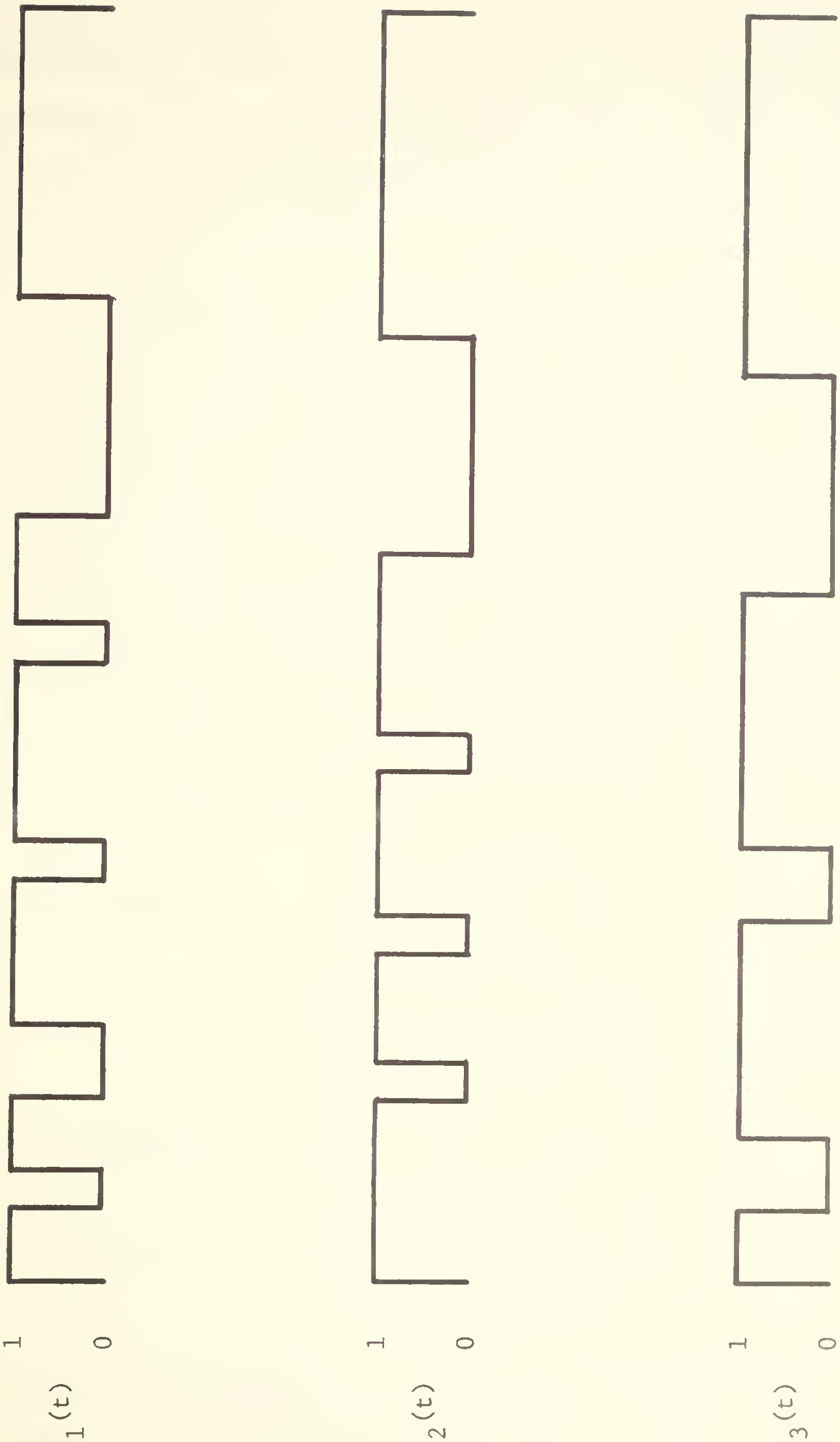
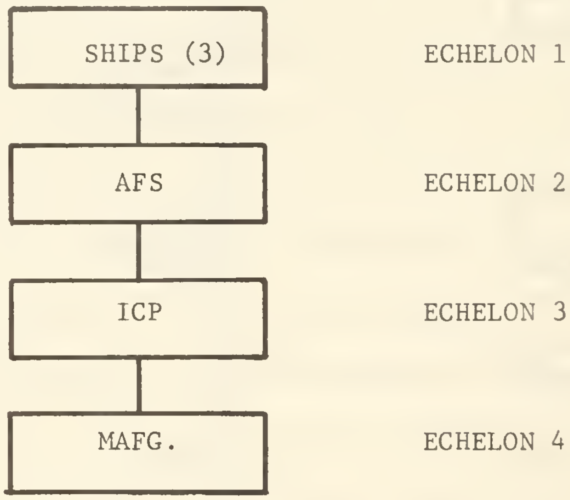


FIGURE 2

analytical solution to the complex Sixth Fleet Supply System made this approach seem infeasible.

A second approach would have been to analyze the data available throughout the Naval Supply System. However, this too presented difficulties for nowhere is data available which traces the movement of a particular requisition through successive echelons until material is supplied. Consequently, a data-oriented study was also excluded.

As a result of the difficulties associated with the two above approaches, a decision was made to study the problem by constructing a computer model to simulate a hypothetical multi-echelon supply system. This simulation model made possible the examination of a wide range of item characteristics as well as a wide variety of system parameters. For simplicity the hypothetical supply system considered in the simulation model has only four echelons. To be precise the system is composed of the following activities:



This hypothetical system eliminates the screening activities and, as with the S^4 simulators, all CONUS stock is treated as though it were concentrated at one conceptual location. Because of the differences between this hypothetical supply system and the actual Sixth Fleet Supply System, the magnitudes of actual supply response times or other quantitative measures may have questionable value; however, it was felt that the hypothesis about the independence of the alternating renewal processes could be tested by studying the simpler supply system.

Every effort was made to make the simulation model of the "reduced" system as realistic as possible. Demands aboard the ships were generated randomly in accordance with instructions in the Ships Supply Support Study. Demand parameters, load lists, reorder levels, reorder quantities and throughput times are all inputs which could be modified easily to accomodate changes. Assumptions about stocking policies, system interactions and material flow followed those assumptions which were incorporated into the Ships Supply Support Study with little deviation.

4. THE SIMULATION MODEL

The modeling of each echelon of the four echelon fleet supply support system is described in the following material, and the fundamental assumptions are indicated.

A. The Fleet (Echelon One)

Demands for a single item are introduced randomly and independently at each of three ships assuming that the times at which demands occur are generated by Poisson processes, and the quantities demanded are geometrically distributed. The dates the demands occur and the amounts demanded are stored to be called in sequence as time advances. The spare stock carried aboard each ship to support the given item is an input parameter which can be determined arbitrarily to give any desired level of protection. If the ship's on-hand stock is sufficient to supply the amount demanded on the date the demand occurs, the requisition is filled immediately and the on-hand stock level is reduced by the appropriate amount. A routine replenishment is forwarded the same day to the support ship (AFS) for the next underway replenishment. The routine replenishment of ship's stock by the AFS occurs in an amount of time which varies between zero and thirty days depending on the physical location of the AFS relative to the ship. If the stock is not available aboard the AFS and the item is a carried item (FILL) the routine resupply aboard the ship is delayed until the AFS stock is itself replenished. If the item is not a FILL item the ship is replenished directly by the ICP.

Those demands which cannot be filled by the ship's storeroom become priority or end-use demands; they are forwarded to either the AFS or the ICP. The decision as to which echelon the requisition is to be submitted is determined by the on-hand stock at the AFS. If the AFS stock is sufficient to fill the demand, the AFS receives the priority requisition; otherwise, the requisition is transmitted to the ICP which in turn sends the material to the ship's home port or some other designated location to be picked up periodically. Priority resupply of the ship occurs in a maximum of eight days from the AFS or in twenty-one days from the ICP whenever stock is available at that echelon. As with routine resupplies, the actual number of days the AFS needs to fill a priority requisition depends on the location of the AFS relative to the position of the ship at the time the requisition is received. In particular, if T is the amount of time to the next ship-AFS rendezvous, the priority requisition leadtime is min [8, T].

B. The AFS (Echelon Two)

Requisitions are submitted to the AFS only if the AFS stock is sufficient to fill the ship's demand. The allotted spare stock depth for the AFS is also an input parameter which can be chosen to correspond to existing requirements such as Fleet Issue Load List (FILL) regulations or may be completely arbitrary. For routine requisitions, units are earmarked for a particular ship and are held until the AFS and the ship rendezvous. As explained earlier, this underway replenishment occurs in a random amount of time determined by the relative positions of the AFS and the ship.

A unit earmarked for routine resupply of a particular ship can be intercepted by another ship only with a higher priority request. When a priority requisition is received by the AFS a search is made first of on-hand stock which is not reserved for other ships, and if the uncommitted stock is not sufficient the "in transit" stock is then searched and reassigned. Whenever a unit which is in transit to one ship is used to fill a priority requisition for another ship, the AFS forwards the ship's routine resupply request to the ICP.

Replenishment stock for the AFS is ordered immediately from the ICP as the on-hand stock is decreased. The AFS is resupplied from the ICP at constant intervals of forty-five days.

C. The ICP and the Manufacturer (Echelons Three and Four)

Routine requisitions received by the ICP are coded as to the originator and filled from stock on hand or backordered as appropriate. The units which are ordered are shipped to storage bins at the ship's home port where they await pickup by the ship or the AFS. Priority requisitions are handled in a similar fashion except that they are sent directly to the ships by expedited means. Shipment time requires twenty-one days from receipt of the request at the ICP to delivery of the material to the ship provided the units are available at the ICP.

The stockage policy for the ICP is a continuous review policy in which a constant amount Q is ordered from the manufacturer each time the inventory position (the stock on hand plus the stock on order less backorders) reaches or falls below a fixed reorder level, R. The manufacturer is assumed to have an availability of one so that every request is satisfied. The delivery from the manufacturer requires 180 days. Upon receipt of units from the manufacturer, priority backorders are filled first and then routine requests are satisfied. Any units remaining after all demands are filled are placed in stock.

D. Input Data

Operation of the simulation model requires only a single card of input to specify the parameters of the demand distributions for each ship and to set the stockage levels for each echelon and the reorder quantity for the ICP. A stockage level of zero will eliminate the AFS or the ships from consideration as a source of supply. Likewise a reorder level and a reorder quantity of zero will eliminate the role of the ICP as a warehouse. In addition to these required input parameters, changes in throughput times and the time period simulated can be made easily. Because of the small amount of required input data and the small computer time required to simulate rather long periods of actual time (runs of three to six years for a single item require only approximately six seconds using the CP-CMS time-sharing system on the Naval Postgraduate School IBM 360/67), the simulation model is quite useful for examining a multitude of questions about the consequences of changes in the supply system.

E. Output Data

All output occurs at the completion of the simulation. For each run an option will give a printout of a day-by-day history of the net inventory at each ship, the AFS and the ICP for the entire length of the simulation. This output allows a requisition to be traced throughout the supply system from its initiation to the final supply action of filling the requisition. Other requisitions which are generated as a result of the initial requisition passing from one level to another can also be followed throughout the system. The cascading effect of a single requisition on supply actions at other echelons can be examined. This day-by-day history is also useful for checking out the model of the supply system. The inclusion of this option adds considerably to the necessary computer time.

Gross supply availability at each echelon is estimated as the ratio of the number of requisitions filled to the number of requisitions received. Similarly, conditional availability at echelon i is estimated as the ratio of the number of requisitions filled at echelon i which could not be filled at any lower echelon to the total number of requisitions received by echelon i which could not be filled at any lower echelon. The gross supply availabilities and the conditional availabilities are both printed out for each echelon. Mean supply response times are calculated using first the gross supply availabilities

A_i, and then the calculations are repeated with the conditional availabilities a_i. These values are included in the output to illustrate the possible impact of any dependence of supply availabilities across echelons.

Finally, values of the test statistics used for the chi-square tests of independence of echelon availabilities, which are described in the ensuing section, are determined and the resulting statistical decisions are presented.

5. THE TESTS FOR INDEPENDENCE

Suppose that n days of supply support operations are simulated and let each day be classified according to three criteria: (1) the stock level of the ship, (2) the stock level of the AFS and (3) the stock level of the ICP. For i = 1, 2 and 3 let

$$Z_i = \begin{cases} 0 & \text{if the on-hand stock at echelon } i \text{ is negative} \\ 1 & \text{otherwise.} \end{cases}$$

Let S_1 be the event $[Z_1 = 1]$ and S_2 the event $[Z_1 = 0]$. Likewise, define A_1, A_2, C_1 and C_2 as the events corresponding to $[Z_2 = 1]$, $[Z_2 = 0]$, $[Z_3 = 1]$ and $[Z_3 = 0]$, respectively.

For purpose of illustration let us restrict our attention to an examination of the relationship between the status of the ship and the status of the AFS. Let n_{ij} be the number of days belonging to the cross classification S_i and A_j for i and j equal to 1 or 2. The n days of simulation can then be partitioned into a 2x2 contingency table as illustrated below:

	A_1	A_2	TOTALS
S_1	n_{11}	n_{12}	$n_{1.}$
S_2	n_{21}	n_{22}	$n_{2.}$
TOTALS	$n_{.1}$	$n_{.2}$	n

To simplify notation the row totals are represented by $n_{i.}$ and the column totals by $n_{.j}$. The goal of this study is to test the null hypothesis that the ship and AFS classifications are independent, that is, the probability that stock is not available at the AFS is not affected by the status of the ship or vice versa. If the classifications are independent then

$$P(A_j \mid S_i) = P(A_j)$$

and

$$P(S_i \mid A_j) = P(S_i)$$

or

$$P(S_i, A_j) = P(S_i)\, P(A_j).$$

Denote the marginal probabilities as follows:

$$P(S_1) = p_1 \qquad P(S_2) = p_2$$

$$P(A_1) = q_1 \qquad P(A_2) = q_2$$

Lastly, denote the joint probabilities, $P(S_i, A_j)$, by P_{ij}. The null hypothesis that the ship and the AFS classifications are independent is equivalent to the null hypothesis:

$$H_o: \; P_{ij} = p_i \, q_j \qquad i = 1, 2 \text{ and } j = 1, 2.$$

Using maximum likelihood estimates, $\hat{P}_{ij} = n_{ij}/n$, $\hat{p}_i = n_{i.}/n$ and $\hat{q}_j = n_{.j}/n$, for the probabilities it is well known that, whenever the null hypothesis is true, the test statistic

$$X^2_{AS} = \sum_{i=1}^{2} \sum_{j=1}^{2} \frac{(n_{ij} - n\hat{p}_i \, \hat{q}_j)^2}{n\hat{p}_i \, \hat{q}_j} \tag{5.1}$$

is approximately chi-square distributed with one degree of freedom for large sample sizes. If the computed value of the test statistic exceeds what could reasonably be expected when sampling from a population having a chi-square distribution with one degree of freedom, one is led to believe that the null hypothesis must be false. Thus, the test of independence of ship and AFS availabilities can be reduced to a simple comparison. Reject the null hypothesis H_o if the computed value of the test statistic X^2_{AS} exceeds $\chi^2_{1-\alpha}(1)$, where $\chi^2_{1-\alpha}(1)$ is that value selected from tables for the chi-square distribution with one degree of freedom which gives the desired significance level α. (The significance level represents the probability of rejecting the null hypothesis when it is indeed true.) Whenever this test results in the rejection of the null hypothesis, the conclusion is that the data present sufficient evidence that the status of the ship depends on the status of the AFS.

In the same manner this test of independence can be repeated for ships versus ICP and the AFS versus ICP. Since the manufacturer is assumed to have an availability of unity, none of the comparisons of a lower echelon with the manufacturer will reject the independence hypothesis.

6. RESULTS

Several computer runs were made using a variety of input parameters to test the validity of the independence assumptions. This was done to insure that resultant output would be obtained for a representative spectrum of item demand characteristics and system stockage levels. In addition, replications were performed looking at the same input parameters with different random numbers, and runs simulating different lengths of time were made. These exercises were undertaken to consider the effect of randomness and to investigate possible transient effects. In some cases there was quite a bit of variability across replications in the magnitudes of the availability estimates and, consequently, great variability in the estimates of mean supply response times. Nevertheless, the general conclusions about independence were consistent.

Sample outputs from the simulation runs are summarized in Table 1, Table 2 and Table 3 for demand rates λ, having respective values of 4/year, 6/year and 12/year at each ship. For each simulation run, the tables display the gross supply availabilities (A_i's), the conditional availabilities (a_i's), the mean supply response times M_1 and M_2 calculated from the gross supply availabilities and the conditional availabilities, the percentage difference in the two estimates of mean supply response time, and the conclusions indicated by the chi-square tests of independence at a significance level of 0.01. The latter conclusions are presented by either an R (for reject H_o) or an A (for do not reject H_o) for each of the chi-square tests for the independence hypotheses concerning ship versus AFS (S/A), ship versus ICP (S/C) and AFS versus ICP (A/C). In order to provide the reader with a reference point, stockage levels and reorder levels for the ships, AFS and

DEMAND RATE = 4/YEAR AT EACH SHIP

PROTECTION LEVEL	STOCKAGE LEVEL SHIP	AFS
(1) 90% PROTECTION FOR 90 DAYS:	2	5
(2) 90% PROTECTION FOR 60 DAYS:	1	4
(3) 90% PROTECTION FOR 30 DAYS:	1	2

ICP REORDER LEVEL	R
(1) 90% PROTECTION FOR LEADTIME:	9
(2) 75% PROTECTION FOR LEADTIME:	8
(3) NOT CARRIED :	0

ICP REORDER QUANTITY

Q = 4

SIGNIFICANCE LEVEL FOR CHI-SQUARE TESTS, $\alpha = 0.01$

STOCK LEVELS			GROSS SUPPLY AVAILABILITIES			CONDITIONAL AVAIL.		MSRT'S		DIFF.	CHI-SQUARE RESULTS		
SHIP	AFS	R	A1	A2	A3	a2	a3	M1	M2	%	S/A	S/C	A/C
2	4	8	1.00	1.00	0.97	xxxx	xxxx	0.5	0.5	0	A	A	A
2	4	9	1.00	0.97	0.95	xxxx	xxxx	0.5	0.5	0	A	A	R
2	4	8	0.98	0.96	0.88	0.75	1.00	0.7	0.7	0	R	R	R
2	5	9	0.97	0.99	0.91	1.00	1.00	0.7	0.7	0	A	R	R
2	0	9	0.97	0	1.00	0	1.00	1.1	1.1	0	A	R	A
2	0	8	0.96	0	0.65	0	0.50	2.5	3.0	20	A	R	A
1	9	9	0.53	0.95	0.28	0.93	0	5.8	7.1	24	R	R	R
1	4	8	0.57	0.81	0.48	0.80	0.10	8.1	10.9	35	R	R	R
1	5	8	0.57	0.85	0.56	0.85	0.43	6.8	7.3	8	R	R	R
1	5	9	0.58	0.81	0.62	0.76	0.33	7.1	10.2	44	R	R	R
1	2	9	0.57	0.56	0.64	0.31	0.51	11.5	19.0	65	R	R	R
1	2	8	0.58	0.56	0.67	0.35	0.60	11.1	16.1	45	R	R	R
1	2	7	0.49	0.54	0.41	0.35	0.25	18.2	28.0	54	R	R	R
1	2	6	0.59	0.57	0.50	0.35	0.18	12.6	23.9	90	R	R	R
1	2	4	0.44	0.52	0.17	0.41	0	26.0	35.4	36	R	R	R
1	1	4	0.59	0.41	0.36	0.19	0.08	19.0	32.2	69	R	R	R
0	5	9	0	1.00	1.00	xxxx	xxxx	8.0	8.0	0	A	A	A
0	4	9	0	0.97	1.00	0.97	1.00	8.4	8.4	0	A	A	A
0	2	9	0	0.67	0.75	0.67	0.63	18.9	22.2	17	A	A	R
0	2	7	0	0.62	0.56	0.62	0.39	25.8	31.0	20	A	A	R
0	0	9	0	0	0.75	0	0.75	40.7	40.7	0	A	A	A
0	0	7	0	0	0.56	0	0.56	55.6	55.6	0	A	A	A
1	0	7	0.60	0	0.56	0	0.37	22.3	28.4	27	A	R	A
1	0	6	0.46	0	0.38	0	0.15	38.4	47.8	24	A	R	A
2	5	0	0.17	0.08	0	0	0	77.3	83.2	8	R	A	A

TABLE 1

DEMAND RATE = 6/YEAR AT EACH SHIP

PROTECTION LEVEL	STOCKAGE LEVEL SHIP	AFS
(1) 90% PROTECTION FOR 90 DAYS:	3	7
(2) 90% PROTECTION FOR 60 DAYS:	2	5
(3) 90% PROTECTION FOR 30 DAYS:	1	3

ICP REORDER LEVEL	R
(1) 90% PROTECTION FOR LEADTIME:	13
(2) 75% PROTECTION FOR LEADTIME:	11
(3) NOT CARRIED :	0

ICP REORDER QUANTITY

Q = 6

SIGNIFICANCE LEVEL FOR CHI-SQUARE TESTS, $\alpha = 0.01$

STOCK LEVELS			GROSS SUPPLY AVAILABILITIES			CONDITIONAL AVAIL.		MSRT'S		DIFF.	CHI-SQUARE RESULTS		
SHIP	AFS	R	A1	A2	A3	a2	a3	M1	M2	%	S/A	S/C	A/C
3	7	13	1.00	1.00	1.00	xxxx	xxxx	0.5	0.5	0	A	A	A
3	0	11	0.99	0	0.99	0	1.00	0.7	0.7	0	A	A	A
2	5	13	0.96	1.00	1.00	xxxx	xxxx	0.8	0.8	0	A	A	A
2	5	11	0.96	1.00	1.00	xxxx	xxxx	0.8	0.8	0	A	A	A
2	0	13	0.93	0	1.00	0	1.00	2.0	2.0	0	A	A	A
2	0	11	0.95	0	0.94	0	0.64	1.6	2.7	69	A	R	A
2	0	11	0.84	0	0.79	0	0.36	6.5	12.0	85	A	R	A
1	7	13	0.50	0.90	0.62	0.90	0	6.3	8.6	38	R	R	R
1	7	13	0.51	0.87	0.48	0.81	0.16	7.7	11.5	49	R	R	R
1	3	11	0.55	0.69	0.67	0.61	0.60	9.3	11.6	25	R	R	R
1	7	11	0.47	0.83	0.53	0.76	0.08	9.1	15.6	71	R	R	R
1	3	11	0.51	0.64	0.61	0.62	0.43	11.8	14.9	26	R	R	R
0	5	11	0	0.90	0.79	0.90	0	11.0	17.5	59	A	A	R
3	7	0	0.17	0.07	0	0	0	77.4	83.1	7	R	A	A
0	0	13	0	0	1.00	0	xxxx	21.0	21.0	0	A	A	A

TABLE 2

DEMAND RATE = 12/YEAR AT EACH SHIP

	PROTECTION LEVEL	STOCKAGE LEVEL SHIP	AFS
(1)	90% PROTECTION FOR 90 DAYS:	5	13
(2)	90% PROTECTION FOR 60 DAYS:	4	9
(3)	90% PROTECTION FOR 30 DAYS:	2	5

	ICP REORDER LEVEL	R
(1)	90% PROTECTION FOR LEADTIME:	24
(2)	75% PROTECTION FOR LEADTIME:	21
(3)	NOT CARRIED	0

ICP REORDER QUANTITY

Q = 12

SIGNIFICANCE LEVEL FOR CHI-SQUARE TESTS, $\alpha = 0.01$

STOCK LEVELS			GROSS SUPPLY AVAILABILITIES			CONDITIONAL AVAIL.		MSRT'S		DIFF.	CHI-SQUARE RESULTS		
SHIP	AFS	R	A1	A2	A3	a2	a3	M1	M2	%	S/A	S/C	A/C
5	13	24	1.00	1.00	1.00	xxxx	xxxx	0.5	0.5	0	A	A	A
4	9	24	1.00	0.99	1.00	xxxx	xxxx	0.5	0.5	0	A	A	A
4	9	21	1.00	0.99	0.99	xxxx	xxxx	0.5	0.5	0	A	A	A
5	13	10	1.00	0.98	0.17	xxxx	xxxx	0.5	0.5	0	A	A	R
2	9	24	0.94	0.99	1.00	1.00	1.00	1.0	1.0	0	R	A	A
2	5	21	0.92	0.90	0.99	0.62	1.00	1.2	1.5	24	R	R	R
2	5	21	0.92	0.86	0.94	0.63	0.73	1.3	2.1	62	R	R	R
2	5	15	0.85	0.70	0.55	0.47	0.33	3.9	7.0	79	R	R	R
2	3	20	0.82	0.56	0.81	0.30	0.64	4.1	7.0	70	R	R	R
2	3	12	0.69	0.51	0.54	0.32	0.38	10.3	15.8	53	R	R	R
1	3	15	0.45	0.55	0.53	0.45	0.36	17.0	24.1	42	R	R	R
1	3	15	0.32	0.43	0.45	0.33	0.37	27.3	34.0	25	R	R	R
3	6	10	0.91	0.79	0.38	0.38	0	2.4	6.5	171	R	R	R
1	6	7	0.16	0.57	0.09	0.51	0	37.5	44.5	19	R	R	R
0	9	21	0	0.99	0.97	0.99	1.00	8.1	8.1	0	A	A	A
0	5	21	0	0.84	0.90	0.84	0.78	11.3	12.8	13	A	A	R
0	5	21	0	0.87	0.97	0.87	0.94	10.0	10.3	3	A	A	R
2	0	21	0.65	0	0.90	0	0.83	10.5	12.4	18	A	R	A
2	0	21	0.38	0	0.97	0	0.96	14.7	15.0	2	A	R	A

TABLE 3

the ICP for various levels of stockout protection are also presented in the tables.

An xxxx entry for the conditional availability of some echelon in a given run indicates that the conditional availability estimate is not meaningful since the gross supply availability of that echelon or some other echelon is unity. When the gross supply availability of a lower echelon is unity the role of each higher echelon to satisfy end-use demands is eliminated. That is, the function of each higher echelon is reduced to that of resupplying the stocks of the lower echelons. Thus, no end-use demands would ever be received at the given echelon. In a similar manner, if the gross supply availability of the given echelon were unity, it automatically follows that the conditional availability must also be unity. For an additional few cases in which the gross supply availabilities are very nearly unity, estimates of the conditional availabilities were determined to be one. However, these estimates were based on such small sample sizes (samples as small as one or two) that the results cannot be meaningful.

Since we are interested primarily with what happens when stockouts do occur, let us ignore those cases described above. In the remaining cases the gross supply availabilities everywhere exceed or are equal to the corresponding conditional availabilites. This in itself is an overwhelming indication of the fallacy of the assumption tested. However, to further support this conclusion, let us examine the results of the chi-square tests of independence.

First, let us test the assumption that the availability of an item at the ship level is independent of the availability of that item at the AFS. In terms of the notation introduced earlier, we are interested in testing the null hypothesis:

$$H_o: \; P_{ij} = p_i \, q_j$$

Before investigating the chi-square tests let us recognize that if stock is always available at either the ship or the AFS ($p_1 = 1$ or $q_1 = 1$) then $P_{ij} = p_i \, q_j$. Similarly, if stock is never available at either the ship or the AFS ($p_2 = 1$ or $q_2 = 1$), then it is also true that $P_{ij} = p_i \, q_j$. Thus, for those cases where the ship has a gross supply availability of zero or one (called trivial cases) it can be argued analytically that the availability of the AFS is independent of the availability of the ship. Likewise, if the AFS has a gross supply availability of zero or one, the availability of the ship is independent of the availability of the AFS. These conclusions are corroborated by the availability figures and the results of the chi-square tests of independence presented in the tables for those cases where gross supply availabilities of the ship or the AFS are either zero or one.

If the trivial cases are filtered out, the chi-square tests reveal that the ship versus AFS independence hypothesis was rejected at a significance level of 0.01 in all but a single case. Furthermore, in that single non-trivial exception, the gross supply availabilities for both echelons were so nearly unity (0.99 and 0.97) that the weak dependence is not surprising. Thus, the chi-square tests of independence indicate that the availability of an item at the first echelon is <u>not</u> independent of the availability of the item at the second echelon in the non-trivial cases.

Similar tests of the independence of the availability of an item at the first and third echelons, as well as at the second and third echelons, all point to the same conclusions. The results of those chi-square tests are also presented in the tables. The conclusions appear to be independent of the demand rates, being a consequence only of the magnitude of the gross supply availabilities. Based on this information it can be concluded that, for the non-trivial cases studied, the item availability at a given echelon depends on the item availability at the other echelons. Obviously, these conclusions can only be made for the particular cases studied. However, because of the wide range of item demand characteristics and stockage levels investigated and the consistency by which the tests reject the independence hypothesis in the non-trivial cases, there is strong evidence that the conclusions might very well be valid when extended to other cases.

In order to display graphically the dependence of the availability of an item at a given echelon and its availability at the other echelons, a sample 900-day history of the in-stock and out-of-stock profile of each echelon is presented in Table 4. In the graphical illustration, a solid line indicates that on-hand stock was greater than or equal to zero and the blank space means that a backorder existed. A solid line occurs at the ship level if and only if no ship had a backorder. The graph shows that runs do occur in which two or three echelons are each simultaneously out of stock. This produces an adverse effect on supply response times. The latter portion of the 900 day period shows that delays at the first echelon are minimal whenever the higher support echelons have stock available. However, when higher echelons are unable to satisfy demands, ships must suffer long delays.

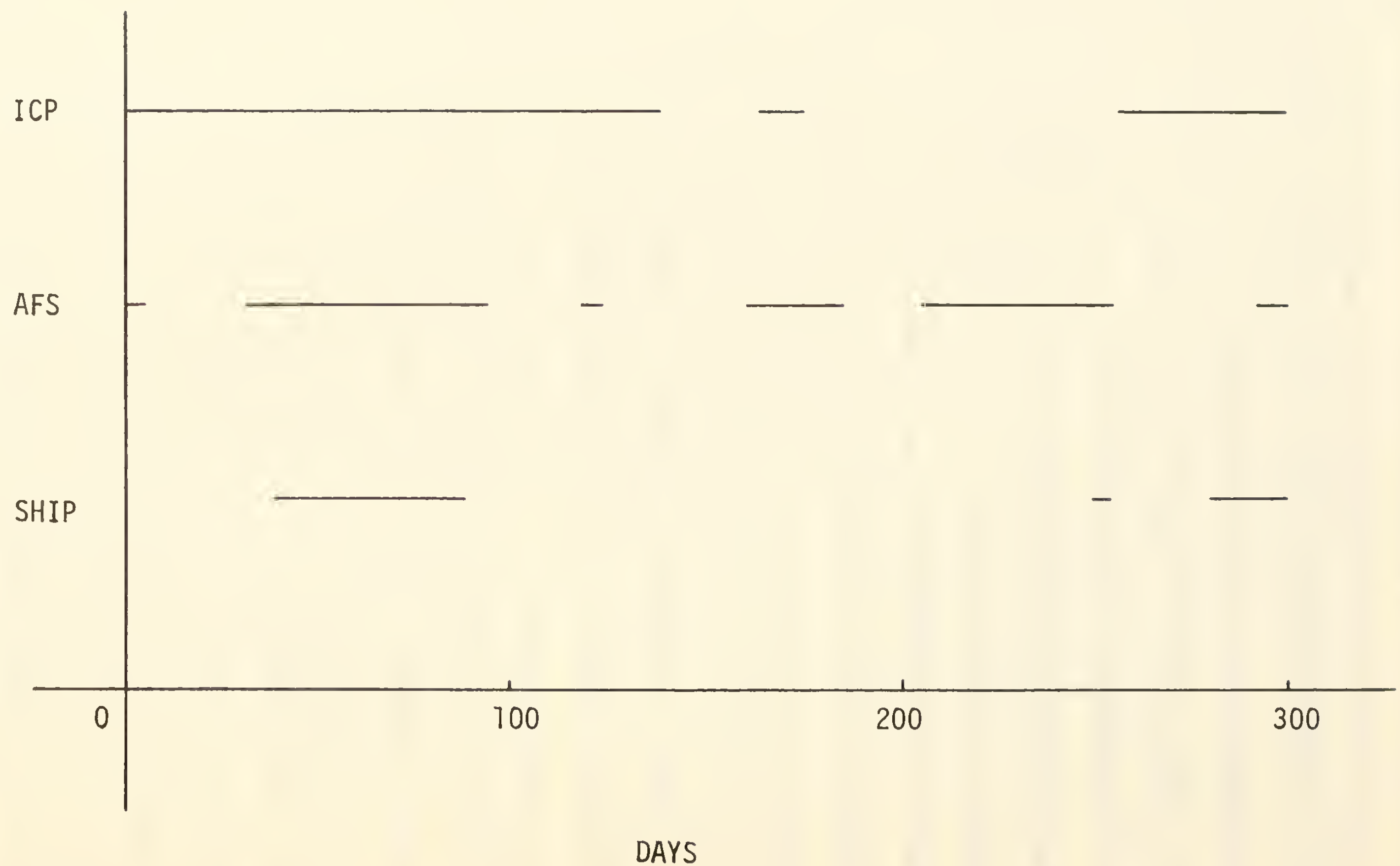


TABLE 4. Stock Profile

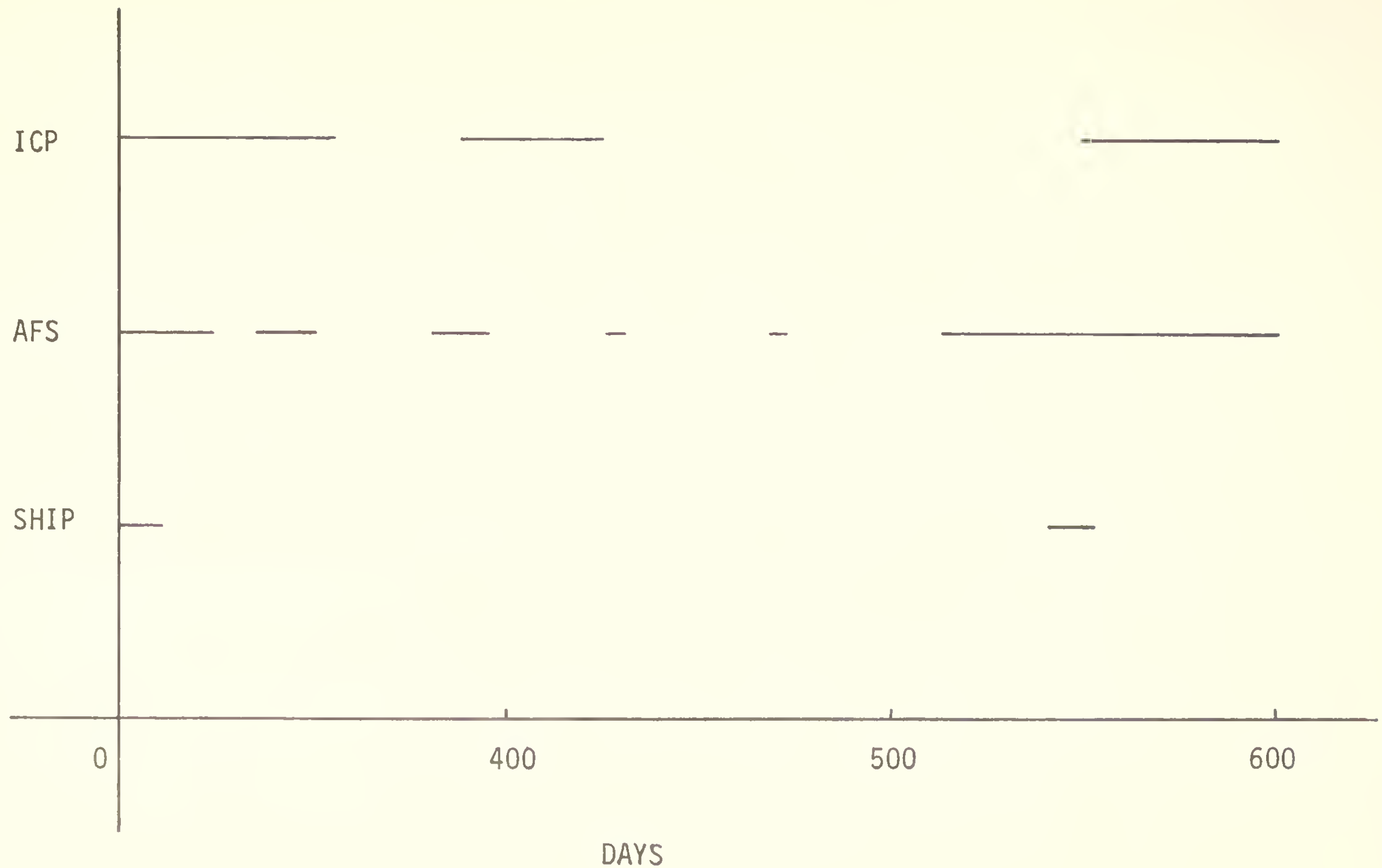


TABLE 4 (Continued). Stock Profile

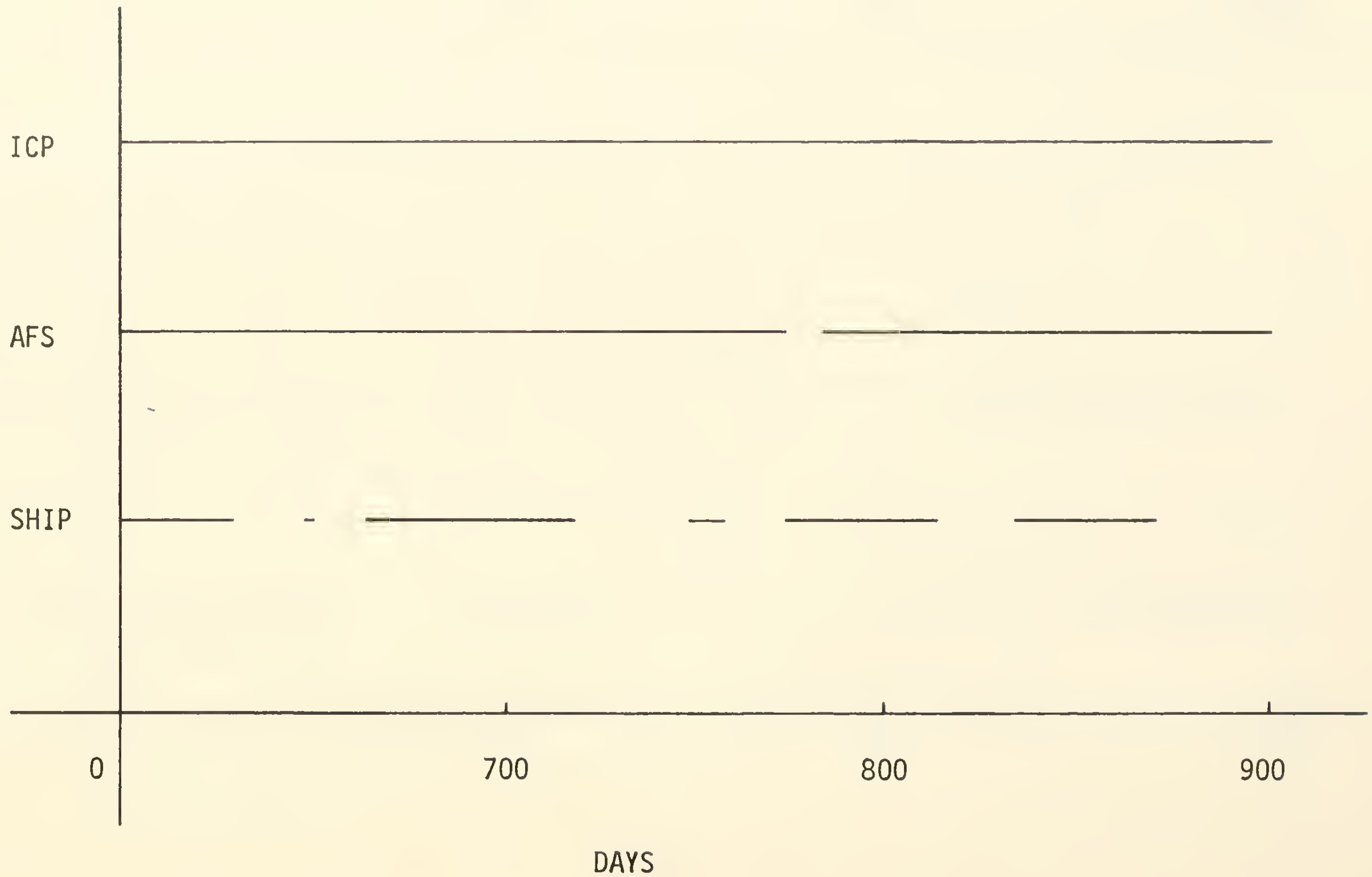


TABLE 4 (Continued). Stock Profile

If the independence hypothesis is rejected it is important to examine the impact of the dependence. To accomplish this, mean supply response times were calculated using, first, the gross supply availabilities in Equation 2.3 for MSRT and then the conditional availabilities. The calculated values of MSRT, M_1 and M_2, along with the percentage differences are shown in Tables 1, 2, and 3. The percentage differences in the sample values for the non-trivial cases can be substantial. In many cases the differences exceed 25%. In all cases, M_1 is less than or equal to M_2. Thus, it would appear from these sample calculations that the Ships Supply Support Study would face a danger of underestimating the true mean supply response time by using gross supply availabilities instead of the conditional availabilities.

7. CONCLUSIONS

The information presented in the preceding section indicates that the availability of an item at a given echelon can depend heavily on its availability at the other echelons, and estimates of mean supply response time in the Ships Supply Support Study can suffer as a consequence. However, for the trivial cases (when availabilities are either zero or one) the item availabilities are independent. Furthermore, the differences, if any, in the two estimates of MSRT were small in the trivial cases as well as in those cases where the gross supply availabilities were nearly unity. In order to evaluate the effect of these conclusions on the Ships Supply Support Study, we must determine the magnitude of the gross supply availabilities actually experienced by the support echelons in the Sixth Fleet. Unfortunately, the gross supply availabilities are required on an item-by-item basis, whereas the data on availabilities is collected and recorded by material cognizance class. Since the cog availability is a weighted average of thousands of item availabilities, it is impossible to decipher item availability figures from a cog availability. One cannot say how a cog availability of, say, .70 translates into item availabilities. On the other hand, some insight as to how the 0.70 figure was obtained can be gained by considering how the range and depth of stock carried at each echelon are determined.

If an item is included in the range of items carried aboard a ship or an AFS as a demand based item, it is carried at a depth sufficient to achieve a prescribed basic combat endurance; generally this is a quantity sufficient to satisfy 90% of the demand during a 90 day period without resupply. Likewise, the reorder level at the stockpoint or the ICP is determined to provide a high level of protection against stockouts. The sample output reveals that gross supply availabilities are very nearly unity whenever stockage levels are set in accordance with the prescribed standards. In fact, the supply availabilities suffer little when the depths at the ship and the AFS are reduced to provide 60 days endurance instead of 90 days. If the item is carried as an insurance item, the availability will probably be nearly unity because demands for insurance items are rare. These arguments tend to indicate that carried items will have availabilities very nearly unity. Obviously, the availability is zero if the item is not carried. Based on these arguments, it seems likely that the cog availability at a given echelon is primarily a weighted average of availabilities of zero and numbers close to unity. Provided this is true, the independence assumption should not critically affect the estimates of mean supply response times.

If the above interpretation given to the cog availability figure is not correct, and a substantial fraction of the items composing the material cognizance class have availabilities which are at neither extreme, then the estimate of cog mean supply response time will probably underestimate the true mean supply response time.

This paper concludes with some remarks about additional possible uses of the computer simulation model. As was pointed out earlier, the output data for the trial runs show that little apparent decline in availability or increase in mean supply response time results from a decrease in stock depth at the ships and the AFS to 60 days endurance from the 90 day endurance level. To make firm conclusions about such a change in COSAL or FILL requirements would require further study. Nevertheless, this points out how the model could be used to investigate the effects of increases or decreases in the stock depth at any level. In the same manner, the model could be used to determine the optimal allocation of a fixed amount of stock throughout the supply system. Furthermore, it could be used to examine the impact of eliminating an echelon completely as a source of supply for a given item or to examine the effect of raising the availability of an item at the stockpoint to a prescribed level.

Supply availability is a function not only of inventory, replenishment rules and demands, but also the resupply times. Thus, the model should be useful for examining the impact of increases or decreases in resupply times. The results obtained in this study of independence used estimates of resupply times based on responses from Sixth Fleet personnel, but they could be modified easily to analyze the sensitivity of mean supply response times to changes in throughput times.

The computer simulation model would seem to offer great potential for providing answers to many questions related to supply support.

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13. ABSTRACT

The Ships Supply Support Study, a study of supply support to the ships of the United States Navy under the direction of the Chief of Naval Operations, assumes that the availability of an item at a given echelon is independent of its availability at other echelons. A study of the validity of that critical assumption is made. A model of a multi-echelon supply support system which provides a history of the daily status of each entity in the system and a stock profile at each echelon is developed. Using randomly generated demands and current replenishment rules for a representative number of items generated at representative activities at each echelon, gross supply availabilities and conditional availabilities are calculated, and statistical tests of the assumption are made. Mean supply response times are computed using both the gross supply availabilities and the conditional availabilities to illustrate the impact of dependence among echelons.

The tests indicate that the availability of an item at a given echelon can in many cases be strongly dependent on its availability at the other echelons. When this is so, the cost in terms of an increase in mean supply response time can be substantial. However, the independence hypothesis is not rejected whenever the gross supply availabilities throughout the system are either very nearly unity or zero.

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14 KEY WORDS	LINK A		LINK B		LINK C	
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